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NUMERICAL SOLUTION OF THE PROBLEM OF HEAT AND MASS TRANSFER IN A MOIST POROUS BODY

A. N. Oblivin and V. S. Kuptsova

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A system of controlling equations is derived. The method of finite differences is used to obtain numerical solutions for the temperature distributions, the moisture content, and the pressure of the air — vapor mixture in a porous body during contact heating.

Heat and mass transfer in a two-dimensional moist porous body during contact heating and molding are discussed. The nonstationary heating of a porous body from a molding surface at constant temperature T_{ms} leads to the vaporization of the moisture in the skeleton and to the formation of an air — vapor mixture in the pores which moves toward the free (permeable) surfaces of the body. The molding process is considered complete when the porous body reaches a given temperature and moisture content.

The motion of the air — vapor mixture in a porous two-dimensional body is described by Darcy's filtration law [1] in the form

$$\Pi\rho u = -k_x \frac{\partial p}{\partial x} \quad \text{and} \quad \Pi\rho v = -k_y \frac{\partial p}{\partial y}, \tag{1}$$

where Π is the volume and surface porosity of the body.

According to the accepted mathematical model of an elementary volume of a porous body shown in Fig. 1, the equation for the transport of the air — vapor mixture can be written in the form

$$\Pi \left(\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{\partial x} + \frac{\partial\rho v}{\partial y} \right) = \beta (p_{sv} - p_v), \tag{2}$$

in which

$$p_v = \rho_v RT, \tag{3}$$

where

$$R = R_v \frac{\rho_v}{\rho} + R_a \left(1 - \frac{\rho_v}{\rho} \right) = R_v \frac{p_v}{p} + R_a \left(1 - \frac{p_v}{p} \right). \tag{4}$$

It is assumed that vaporization and condensation of moisture occur at the pore surfaces and that the moisture in the system of capillary channels of the skeleton is in the liquid state at the skeleton temperature T_{sk}. It is assumed that the mass transfer rate in the bulk of the body is proportional to the difference between the saturated vapor pressure at the temperature T_{sk} and the partial pressure of the vapor p_v.

The moisture content in the skeleton is characterized by W, and its local time rate of change is described by the moisture content equation

$$(1 - \Pi) \frac{\partial W}{\partial t} = \beta (p_v - p_{sv}). \tag{5}$$

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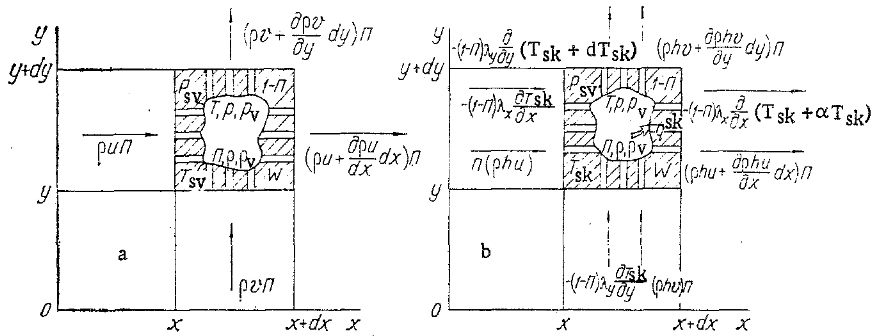

Fig. 1. Model of matter (a) and heat transfer (b) in a porous body.

The equation for the transport of vapor in the pore system is similar to Eq. (2) and has the form

$$\Pi\left(\frac{\partial \rho_v}{\partial t}+\frac{\partial \rho_v u}{\partial x}+\frac{\partial \rho_v v}{\partial y}\right)=\beta\,(p_{sv}-p_v).$$

Using Eq. (2) this equation can be rewritten in the form

$$\Pi\left(\rho\frac{\partial \bar p}{\partial t}+\rho u\,\frac{\partial \bar p}{\partial x}+\rho v\,\frac{\partial \bar p}{\partial y}\right)=(1-\bar p)\,\beta\,(p_{sv}-p_v), \tag{6}$$

where $\bar p = \rho_v/\rho = p_v/p$ is the dimensionless relative partial pressure of the vapor in the mixture.

We return to the model of an elementary volume of a porous body (Fig. 1b) to describe the heat-transfer process. Heat transfer in the skeleton requires taking account of conduction and the thermal effects accompanying the vaporization and condensation processes on the pore surfaces and is described by the equation

$$(1-\Pi)\,\frac{\partial}{\partial t}\,(c_{sk}\rho_{sk}+c_{mo}W)\,T_{sk}=(1-\Pi)\left(\lambda_x\,\frac{\partial^2 T_{sk}}{\partial x^2}+\lambda_y\,\frac{\partial^2 T_{sk}}{\partial y^2}\right)+q^{sk}. \tag{7}$$

Here q^{sk} is the total thermal effect of the interaction of the air – vapor mixture and the skeleton, and it can be interpreted as the volumetric heat release rate consisting of the four components

$$q^{sk}=q_1^{sk}+q_2^{sk}+q_3^{sk}+q_4^{sk}, \tag{8}$$

where

$$q_1^{sk}=\alpha\,(T-T_{sk}) \tag{9}$$

is the volumetric heat flux density due to convective heat transfer at the pore surfaces, where α is the volumetric heat-transfer coefficient; it depends on the mass flux ρw through the pores, where

$$w=\sqrt{u^2+v^2}\,, \tag{10}$$

$$q_2^{sk}=\beta\,(p_v-p_{sv})\,r \tag{11}$$

is the heat release rate at the pore surfaces resulting from the condensation of vapor (for evaporation when $p_{sv} > p_v$, $q_2^{sk} < 0$; i.e., the skeleton loses energy);

$$q_3^{sk}=c_v\beta\,(T-T_{sk})\,\frac{1}{2}\,(p_v-p_{sv}+|\,p_v-p_{sv}\,|) \tag{12}$$

is the heat release rate from the cooling of vapor from the temperature T of the mixture to the temperature T_{sk} of the skeleton before condensing on the pore surfaces. If $T < T_{sk}$, during condensation $q_3^{sk} < 0$ and heat is absorbed by the vapor before condensing; if there is no condensation $p_v \le p_{sv}$ and $q_3^{sk} = 0$;

$$q_4^{sk}=\beta\,(p_v-p_{sv})\,c_{mo}T_{sk} \tag{13}$$

is the rate of heat input to the skeleton (or output for $p_v < p_{sv}$) as a result of the increase (decrease) of the heat content during the increase (decrease) of the moisture content of the skeleton.

After substituting Eqs. (8), (9), (11), (12), and (13) into the right-hand side of Eq. (7), using (5), and making some transformations, we obtain

$$(1-\Pi)(c_{sk}\rho_{sk}+c_m\delta W)\frac{\partial T_{sk}}{\partial t}=(1-\Pi)\left(\lambda_x\frac{\partial^2 T_{sk}}{\partial x^2}+\lambda_y\frac{\partial^2 T_{sk}}{\partial y^2}\right)+$$
$$+\alpha(T-T_{sk})+\beta(p_v-p_{sv})r+c_v\beta(T-T_{sk})\frac{1}{2}(p_v-p_{sv}+|p_v-p_{sv}|). \quad (14)$$

Heat transfer in the pores is determined by taking account of convection and the thermal effects accompanying vaporization and condensation on the pore surfaces. If the available heat per unit volume of the air — vapor mixture is characterized by the heat content h, the heat-transfer equation for the mixture takes the form

$$\Pi\left(\frac{\partial\rho h}{\partial t}+\frac{\partial\rho hu}{\partial x}+\frac{\partial\rho hv}{\partial y}\right)=q. \quad (15)$$

The overall thermal effect of the interaction of the skeleton and the air — vapor mixture q can be separated into the following three components:

$$q=q_1+q_2+q_3, \quad (16)$$

where

$$q_1=\alpha(T_{sk}-T) \quad (17)$$

is the volumetric heat flux density due to convective heat transfer at the pore surfaces:

$$q_2=c_v\beta(T_{sk}-T)\frac{1}{2}(p_{sv}-p_v+|p_{sv}-p_v|) \quad (18)$$

is the heat release rate resulting from the cooling of vapor from the skeleton temperature T_{sk} to the temperature of the mixture T after vaporization at the pore surfaces (if $T > T_{sk}$ during vaporization $q_2 < 0$; i.e., the mixture transfers energy to the heating of the vapor from the vaporization temperature T_{sk} to the temperature T of the mixture, and when there is no vaporization $p_v \geq p_{sv}$ and $q_2 \equiv 0$);

$$q_3=\beta(p_{sv}-p_v)h_v \quad (19)$$

is the rate of heat input to the mixture (or loss if $p_{sv} < p_v$) as a result of mass transfer during vaporization (or condensation) of moisture on the pore surfaces, where h_v is the heat content of the vapor at the temperature of the mixture T.

After substituting Eqs. (16)-(19) into the right-hand side of (15) and using (2), we obtain

$$\Pi\left(\rho\frac{\partial h}{\partial t}+\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial u}\right)=\alpha(T_{sk}-T)+\beta(p_{sv}-p_v)(h_v-h)+c_v\beta(T_{sk}-T)\frac{1}{2}(p_{sv}-p_v+|p_{sv}-p_v|). \quad (20)$$

Using the fact that

$$h=cT+\bar{p}r,\ h_v=c_vT+r \text{ and } c=c_v\bar{p}+c_a(1-\bar{p}),$$

and Eq. (6), we can write Eq. (20) in the form

$$\Pi c\left(\rho\frac{\partial T}{\partial t}+\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial y}\right)=\alpha(T_{sk}-T)+c_v\beta(T_{sk}-T)\frac{1}{2}(p_{sv}-p_v+|p_{sv}-p_v|). \quad (21)$$

Thus, the mathematical formulation of the problem under consideration includes seven partial differential equations and four algebraic equations containing the 11 unknowns ρ, p, $\bar{p}$, T, T_{sk}, W, ρu, ρv, R, c, and p_{sv}:

the filtration equation,

$$\Pi\rho u=-k_x\frac{\partial p}{\partial x},\quad \Pi\rho v=-k_y\frac{\partial p}{\partial y};$$

the transport equation for the air — vapor mixture,

$$\Pi\left(\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}\right)=\beta(p_{sv}-p\bar{p});$$

the moisture content equation

$$(1-\Pi)\frac{\partial W}{\partial t}=\beta(p\bar{p}-p_{sv});$$

the vapor transport equation,

$$\Pi\left(\rho\frac{\partial\bar{p}}{\partial t}+\rho u\,\frac{\partial\bar{p}}{\partial x}+\rho v\,\frac{\partial\bar{p}}{\partial y}\right)=(1-\bar{p})\,\beta\,(p_{sv}-p\bar{p});$$

the heat-transfer equation in the skeleton,

$$(1-\Pi)\,(c_{sk}\rho_{sk}+c_{mo}W)\,\frac{\partial T_{sk}}{\partial t}=(1-\Pi)\left(\lambda_x\,\frac{\partial^2 T_{sk}}{\partial x^2}+\lambda_y\,\frac{\partial^2 T_{sk}}{\partial y^2}\right)+$$

$$+\alpha(T-T_{sk})+\beta(p\bar{p}-p_{sv})\,r+c_v\beta(T-T_{sk})\,\frac{1}{2}\,(p\bar{p}-p_{sv}+|p\bar{p}-p_{sv}|);$$

the heat-transfer equation for the air — vapor mixture,

$$\Pi c\left(\rho\,\frac{\partial T}{\partial t}+\rho u\,\frac{\partial T}{\partial x}+\rho v\,\frac{\partial T}{\partial y}\right)=\alpha(T_{sk}-T)+$$

$$+c_v\beta(T_{sk}-T)\,\frac{1}{2}\,(p_{sv}-p\bar{p}+|p_{sv}-p\bar{p}|);$$

the algebraic relations

$$R=R_v\bar{p}+R_a(1-\bar{p}),\quad c=c_v\bar{p}+c_a(1-\bar{p}),$$
$$p=\rho RT,\quad p_{sv}=f(T_{sk}).$$

It should be noted that the equation for the overall heat transfer in a porous body obtained by a term-by-term addition of the equations for heat transfer in the skeleton and in the air — vapor mixture.

$$(1-\Pi)\,(c_{sk}\rho_{sk}+c_{mo}W)\,\frac{\partial T_{sk}}{\partial t}+\Pi\rho c\,\frac{\partial T}{\partial t}=$$

$$=(1-\Pi)\left(\lambda_x\,\frac{\partial^2 T_{sk}}{\partial x^2}+\lambda_y\,\frac{\partial^2 T_{sk}}{\partial y^2}\right)-\Pi c\left(\rho u\,\frac{\partial T}{\partial x}+\rho v\,\frac{\partial T}{\partial y}\right)+$$

$$+\beta(p\bar{p}-p_{sv})\,[c_v(T-T_{sk})+r]$$

shows the validity of taking account of all the thermal effects in the overall energy balance. The last term on the right-hand side of this equation characterizes the heat release during cooling of the condensed vapor from the temperature of the mixture T to the temperature of the skeleton T_{sk} and the subsequent condensation at this temperature or the available heat during the inverse process.

It was assumed that at t = 0 the quantities sought were uniformly distributed through the porous body:

$$p=p_0,\quad \bar{p}=\bar{p}_0,\quad T_{sk}=T=T_0,\quad W=W_0. \tag{22}$$

The problem was solved for thermal boundary conditions of the first kind on the molding surfaces of the porous body ($T_{sk}=T_{ms}$). The velocity of the mixture in the direction of the normal n to the impermeable surface of the body (subscript is) is equal to zero ($w_n/_{is}=0$). Consequently,

$$\left.\frac{\partial p}{\partial n}\right|_{is}=0.$$

The boundary conditions on the impermeable surface of the body for the vapor and heat-transfer equations in the air — vapor mixture take the form of degenerate variants of these equations:

$$\Pi\left.\left(\rho\frac{\partial\bar{p}}{\partial t}+\rho w_\tau\,\frac{\partial\bar{p}}{\partial\tau}\right)\right|_{is}=(1-\bar{p}_{is})\,\beta\,(p_{sv}-p_{sv}\bar{p}_{sv}) \tag{23}$$

and

$$\Pi c\left.\left(\rho\,\frac{\partial T}{\partial t}+\rho w_\tau\,\frac{\partial T}{\partial\tau}\right)\right|_{is}=\tilde{\alpha}_{is}\,(T_{sk}-T)|_{sv}, \tag{24}$$

where w_τ is the velocity of the mixture in the tangential direction τ, at right angles to the direction of the normal n;

$$\tilde{\alpha}_{is}=\alpha_{is}+c_v\beta\,\frac{1}{2}\,(p_{sv}-p_{is}\,\bar{p}_{is}+|p_{is}-p_{is}\bar{p}_{is}|)$$

is the reduced volumetric heat-transfer coefficient calculated from the conditions on the impermeable surface.

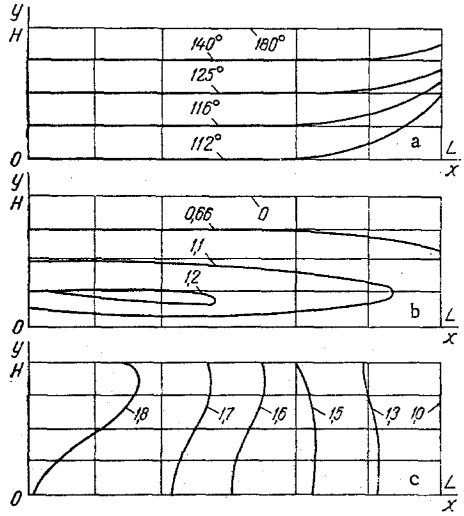

Fig. 2. Distribution of: a) skeleton temperature T_{sk}; b) moisture content W; c) pressure p for Fo = 0.8 (temperature is in °C, pressure is in atm, and the moisture content is relative to the initial value).

For the symmetry plane of a porous body (subscript sp) with the direction of the normal n we have

$$\left.\frac{\partial p}{\partial n}\right|_{sp} = \left.\frac{\partial \bar{p}}{\partial n}\right|_{sp} = \left.\frac{\partial T}{\partial n}\right|_{sp} = \left.\frac{\partial T_{sk}}{\partial n}\right|_{sp} = 0. \tag{25}$$

For a free permeable surface of a body (subscript ps) with the direction of the outward normal n the pressure p of the air — vapor mixture must be equal to the pressure of the surrounding medium p_m:

$$p_{ps} = p_m. \tag{26}$$

This condition leads to the vanishing of the mass flux of the mixture along a permeable surface. The boundary condition for the temperature of the skeleton in this case can be written in the form

$$\lambda_n \left.\frac{\partial T_{sk}}{\partial n}\right|_{ps} = \alpha_{ps}(T_m - T_{sk})|_{ps}, \tag{27}$$

where λ_n is the thermal conductivity of the skeleton in the direction of the normal, and α_{ps} and T_m are the heat-transfer coefficient and the temperature of the medium at the permeable surfaces.

The boundary condition for the vapor transport equation is determined by the mass-transfer characteristics at the permeable surface and can be written in the form

$$\rho w_n \bar{p}|_{ps} = \beta_{ps}(\bar{p}_{ps} - \bar{p}_m)p_a \tag{28}$$

where w_n is the velocity of the mixture in the direction of the normal to the permeable surface. Since according to Eq. (1)

$$\Pi \rho w_n|_{ps} = -k_n \left.\frac{\partial p}{\partial n}\right|_{ps}, \tag{29}$$

we can rewrite Eq. (29) in the form

$$\frac{k_n}{\Pi}\bar{p}_{ps}\left.\frac{\partial p}{\partial n}\right|_{ps} = \beta_m(\bar{p}_m - \bar{p}_{ps})p_m, \tag{30}$$

where k_n is the permeability of the porous body in the direction of the normal to the permeable surface. The boundary condition for the heat-transfer equation in the air — vapor mixture at this surface is

$$\Pi c \rho w_n \left.\frac{\partial T}{\partial n}\right|_{ps} = \tilde{\alpha}_{ps}(T_{sk} - T)|_{ps}. \tag{31}$$

where

$$\tilde{\alpha}_{ps} = \alpha_{ps} + c_v\beta \frac{1}{2} (p_{sv} - p_m\bar{p}_{ps} + |p_{sv} - p_m)\bar{p}_{ps}|)$$

is the reduced volumetric heat-transfer coefficient calculated from conditions at a permeable surface of the porous body.

The problem formulated cannot be solved analytically. Therefore, we solved the problem by the finite-difference method using an explicit scheme for all equations of the evolutionary type [2]. An analysis of the stability of the numerical solution and the convergence of the finite-difference scheme used gave satisfactory results. The algorithm developed for the numerical solution was programmed in ALGOL-60 for a BÉSM-6 computer.

The numerical analysis led to a rather complete picture of the time development of the heat- and mass-transfer processes over large ranges of the controlling parameters. The nonstationary temperature distributions of the skeleton and air — vapor mixture, the moisture content, and the pressure in the molding process were obtained. As an example, Fig. 2 shows the temperature distribution in the skeleton, the pressure of the mixture, and the moisture content in a porous body (chip board) for $Fo = \lambda_y t/(c_{sk}\rho_{sk}H^2) = 0.8$, where H is the half-thickness of the board.

Detailed information on the parameters of the process obtained by a numerical solution of the problem makes it possible not only to judge the state of the body at any instant, but also to optimize technological procedures for molding porous bodies.

NOTATION

x, y, coordinates; Π, porosity; ρ, density; p, pressure; T, temperature; u, v, x, y, components of velocity of air — vapor mixture; t, time; k, permeability; λ, thermal conductivity of skeleton; c, specific heat; R, gas constant; W, moisture content; h, heat content; r, heat of vaporization; q, volumetric heat-release rate; α, volumetric heat-transfer coefficient; β, volumetric mass-transfer coefficient; $\bar{p}$, dimensionless relative partial pressure of vapor; n, direction of normal to surface; τ, tangential direction; H, half-thickness of board; L, half-width of board; Fo, Fourier number; w, velocity of air — vapor mixture. Indices: a, air; v, vapor; sk, skeleton; m, surrounding medium; sv, saturated vapor; mo, moisture; n, direction of normal to surface; τ, tangential direction; 0, zero time; ms, molding surface, is, impermeable surface; sp, symmetry plane of porous body, ps, permeable surface of porous body.

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